# System of Differential Equations

To control this process, differential equations that describe the state of the process at a point in time are derived. Although the process in question is a PFR, it can be simulated through assuming several CSTRs in series. For the first set of differential equations, a material balance was performed on 5 CSTRs where the output of the first CSTR feeds into the input of the second etc... These equations are as follows:

Equation 1)

(Equation 2)

Equation 3)

Equation 4)

Equation 5)

To find the concentration leaving the CSTRs at any point any time, Euler’s method was applied to the equations to estimate their values at a time t:

Equation 6

(Equation 7)

Equation 8

Equation 9

Equation 10

Next, the transfer function for the measuring device was obtained by applying the constraints provided in the problem statement:

Equation 11

This equation was then converted into the time domain as follows:

Equation 12

Similar to the previous equations, Euler’s method was applied again to get the following:

Equation 13

The next step was to find the equation describing the error, which is as follows:

Equation 14

Next a discrete velocity form of the PID equation was obtained in order to be able to manipulate the equation in the time domain:

Equation 15

Finally, an equation governing the new flowrate obtained by manipulating the opening of the valve was obtained. Kv was calculated by dividing the maximum flowrate allowed by the valve and dividing it by the maximum and minimum pressure that can be outputted:

(Equation 16)

# Optimization Parameters for the PID controller

To achieve the best optimization for the PID controller, there exists certain parameters that relate to the error function that can be used to make the controller as efficient as possible. The methodology here is to attempt and make these parameters as small as possible to reduce error in the controller response thus allowing steady state to be reached faster. These parameters are minimized through the manipulation of the controller parameters Kc, τI, and τD. The optimization parameters will be discussed as follows:

1. IAE criterion: Integral absolute error

This error is described by . This error describes the area underneath the curve obtained by the control system.

1. ISE criterion: Integral of the squared error

This error is described by This error results in a rapid rise time however manipulating this criterion will result in a more oscillatory response.

1. ITAE criterion: Integral time weighted squared error

This error is described by . This error penalizes errors that occur late in time and thus produces a smoother response.

# Design and Performance of the PID controller

As stated previously, the process was modelled after several CSTRs placed in series to simulate a plug flow reactor. The initial conditions chosen were when the volumetric flow rate was equal to 2 m3/s with an initial concentration of 10 mol/m3 as stated by the design problem. The valve was also assumed to be completely closed at the initial point. Five CSTRs were chosen to be places in series and all their volumes were taken as equal such that every CSTR was 0.6 m3 in size. A step size of 0.01 min was chosen for the calculation of all the concentrations, flowrates, etc…

Initially, the system was simulated with no disturbances for a total of 10 minutes until it reached steady state. This was followed by an increase in the initial concentration coming in from the feed to 12 mol/m3 for another 10 minutes where then the concentration was reduced to 8 mol/m3 for a final 10 minutes. The system was then restored to its initial concentration and allowed to run for a final 10 minutes.

A second set of disturbances were introduced to the system; however, these were changes to the setpoint. Initially the system was allowed to run under the initial setpoint of 2.5 mol/m3for 10 minutes until it reaches steady state. Immediately after, the set point was changed to 3.5 mol/m3 for a total of 10 minutes. The setpoint was then again reduced to 1.5 mol/m3 for 10 minutes. Finally, the system was restored to its initial conditions and the system was allowed to run for a final 10 minutes.

As discussed before, the optimization parameters were observed during the disturbances and setpoint changes to properly optimize the controller parameters.

# Results and Analysis

After trial and error in attempting to minimize the error post-disturbance, the suggested PID controller parameters are:

1. Kc = 0.6
2. τI= 0.1
3. τD= 0.05

**Disturbance of initial concentration:**



Figure 1. Variation of PFR output concentration as a function of time

During the disturbance of the feed concentration, it can be observed that there is a delayed peak visible. This is due to the transport lag and is expected of a PFR. In addition, the system was quick to stabilize after reaching steady state once. To first reach steady state, it took around 6 minutes. However, after each disturbance, the time required to stabilize was close to 3 minutes which speaks to the efficiency of the PID controller.

**Change of initial setpoint:**



Figure 2. Variation of PFR output concentration as a function of time

The same delayed peaks can be observed for the change in initial setpoint which is again a result of the transport lag. Similar results can be observed in this situation in comparison to the disturbance of concentration. The system took longer to reach steady state after the first run and reached it much faster after every setpoint change.

Table 1. Comparison of optimization parameters under different conditions

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | No Disturbance | Setpoint Change | Concentration Disturbance |
| IAE | 15.2923 | 34.71067 | 20.5875 |
| ISE | 22.1519 | 35.08159 | 22.6135 |
| ITAE | 10.487 | 416.286 | 132.591 |

The optimization parameters for the system were very promising. The variation of all the parameters is not large when comparing initial conditions, setpoint change, and concentration disturbance. This again proves the efficiency of the PID controller by reducing the errors so minimally.

Although the PID controller is efficient under the problem statement conditions, it does have limitations. For instance, the initial feed concentration cannot drop beyond 4 mol/m3. If this happens, the required flowrate to achieve steady state will be greater than the maximum flowrate achievable by the valve. In this situation for instance, steady state will not be achieved. Under the same premise, the output concentration from the PFR cannot exceed 5.5 mol/m3 because it would again require a flowrate greater than what the valve can provide.

Furthermore, the reaction order plays a large role in the stability of this control system. This system was modelled through an approximation at a time t using Euler’s method. Although Euler’s method is easy and fast, it only works for 1st order systems. In this case, increasing the reaction order 2 will result in inaccurate information and never reaching steady state. This causes the system not to be robust in face of higher order reactions. This problem can be avoided by using higher order approximation techniques such as Runge-Kutta for example. When changing the reaction order from 1 to 2, the calculated error parameters increase 2-fold, showing large sensitivity. However, the system is somewhat resilient to changes in the reaction constant where it was increased by 4-fold and the system still maintaining stability through slight adjustments of the PID controller parameters.

The dynamics of the sensor and actuator do not adversely affect the performance of the controller. Experiments decreasing the time constants for both devices resulted in very marginal improvement in the performance of the controller. So, it’s safe to say that their dynamics currently don’t have a negative effect on the system. However, if the device parameters were to increase, and keeping the parameters of the PID controller the same, the system could be thrown into instability. Therefore, although lower time constants don’t affect the system greatly, it’s not suggested to use devices with time constants higher than what is currently being used.